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FREE INTERNAL WAVES IN ICE-COVERED SEAS (O SVOBODYNKH VNUTRENNI--ETC(U)
SEP 77 V N SMIRNOV, V G SAVCHENKO

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FREE INTERNAL WAVES IN ICE-COVERED SEAS

(Smirnov, V. N. and V. G. Savchenko, O Svobodynkh vnutrennikh volnakh v more, pokrytom l'dom, Arkt. i antarkt. n.-i. inst., Trudy No. 306, 1972, Russian)

A great deal of attention has recently been paid to studying surface /108
waves in ice-covered seas. The work by K. Hukins (15) theoretically examined the dispersion curves of surface elastic gravity waves in a 0.1 - 0.0 sec period range. Instrument observations in the Arctic Ocean and in Antarctica have made it possible to investigate waves nearly throughout this entire range (4, 7, 15, 16). It was established that natural oscillations of the ice cover are chiefly due to hummocking of the ice, the effect of wind, and the passage of free gravity waves from the clear water. The recorded wave parameters and the theoretical concepts agree well.

In 1970 oscillations of the ice cover that did not obey the laws of propagation of surface elastic gravity waves were recorded at the "Severnyy polyus-20" (North Pole 20) drift station. The results of the investigations enable one to hypothesize that these oscillations are a manifestation of internal dynamic processes that develop in the ocean depths.

The observations were made on a pack ice floe about 3.8 X 4.3 km in size. Its thickness averaged 4.5 m. The ice floe was made up of ice fields frozen together. Polynias (translator's note: areas of open water in ice) and open areas of water formed in the Summer between the ice fields. The installation for observing the ice cover oscillations was set up 1.5 km from the edge of the ice floe.

The ice cover oscillation sensors were seismic tilt meters - horizontal pendulums that react both to elastic waves and ice surface inclinations. The pendulum displacements were recorded on moving recorder tape. The layout of the tilt meters and the method of making observations with them are presented in a work (8).

A total of four identical tilt meters were set up on the ice cover. Two instruments with mutually perpendicular orientation were set up in one point and the others were set up 25 m apart on a straight line. This arrangement of the instruments made it possible to determine the velocity /109
and direction of wave propagation. Oscillations from one of the sensors were continuously single-channel recorded on the N-384 type automatic ampere voltmeter; oscillations from all four sensors were simultaneously recorded periodically on the tape of the N-700 oscillograph. The frequency characteristic of the recording channel in the observable range of periods was rectilinear and the maximum sensitivity to tilts was 0.001 sec. arc/mm.

The amplitude ε of the vertical oscillations of the ice cover was determined according to the formula

$$\varepsilon = 2\xi = \frac{\lambda\psi}{\pi}$$

where ξ is wave amplitude;

λ is wavelength;

ψ is the maximum inclination of the floe during passage of the wave[1].

Figure 1 gives typical recordings of ice cover oscillations. Relatively short period (τ = 20 - 25 sec) waves of the swell type are superimposed on the long period oscillations. Vector diagrams which are a polarization of the long period waves in the floe plane (Figure 2) were plotted. Polarization of the oscillations shows a quite stable progressive wave with a certain direction of propagation: the azimuth of each wave remains constant for the entire time of its existence. This circumstance enables one to assert that wave motion has been recorded.

The histogram of wave periods (Figure 3) was plotted for the half-year cycle of continuous observations. About 70% of the 1,120 measured values of periods are in the 6 - 13-minute range.

Table 1 gives the values of parameters of the most stable groups of waves recorded at different times using positional-azimuthal arrangement of the tilt meters on the pack ice floe.

Table 1

Values of Periods T, Phase Velocities v, Length λ, Azimuths α, and Amplitudes ε of Waves

Example	Date	T, sec	v, m/sec	λ, м	α	ε, мм
	1970					
1	11/VII	840	1,6	1340	57°	4,00
2	27/VIII	660	1,1	730	26	4,36
3	28/VIII	880	1,1	980	41	4,90
4	28/VIII	800	1,2	960	36	4,33
5	28/VIII	900	1,0	900	180	3,12
6	28/VIII	1080	0,7	760	180	2,66
7	14/IX	700	2,0	1400	50	4,20
8	14/IX	670	0,5	330	303	0,84
9	14/IX	670	0,6	400	300	1,00

[1] The highest recorded value of ψ was 4", 0, and that of ε - 6.0 mm.

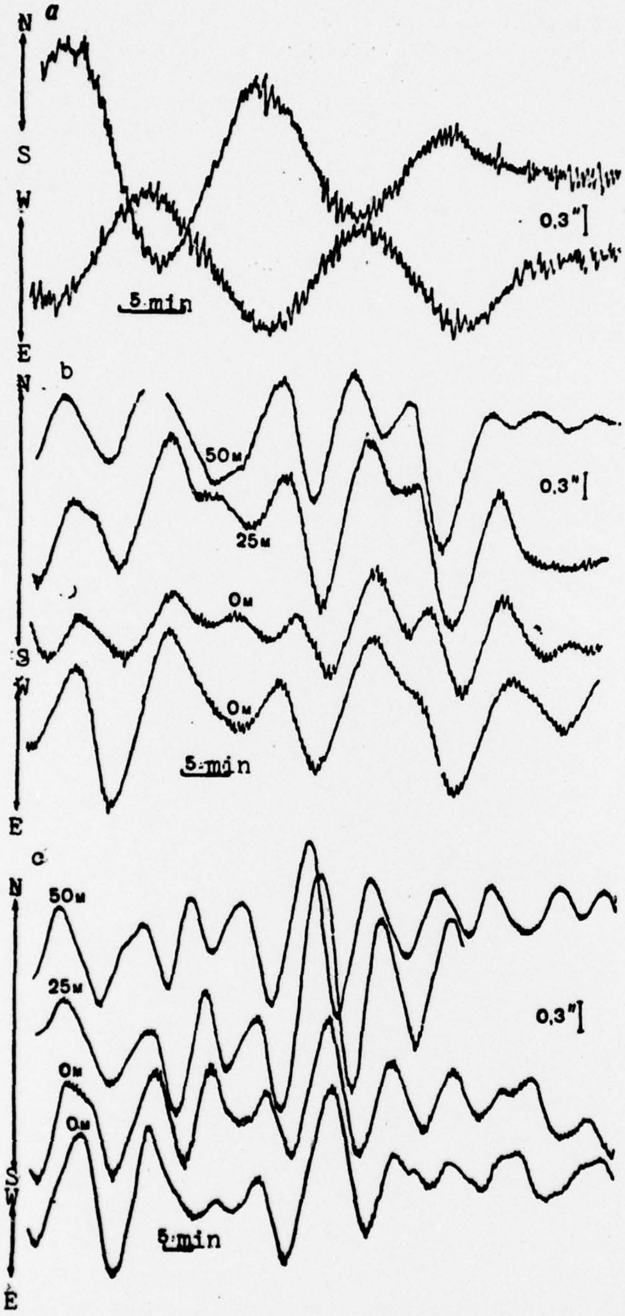

Figure 1. Typical recordings of waves obtained on a pack ice floe using two tilt meters with mutually perpendicular orientation (a) and positional-azimuthal arrangement of tilt meters (b, c).

All calculations were made with the assumption that floe drift velocity is zero, e.g., the Doppler effect was not taken into account. /111

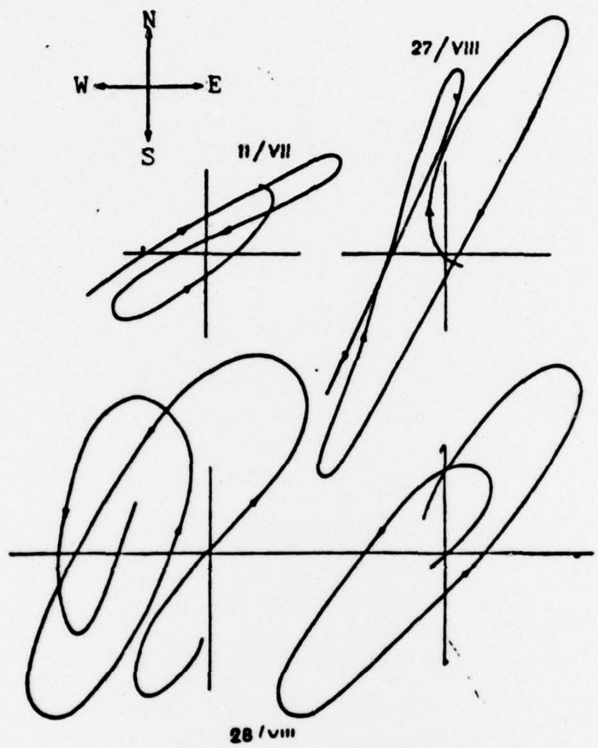

Figure 2. Polarization of waves in the horizontal plane for separate series of observations.

The observation results enable one to hypothesize that the appearance of such slow waves is due to the subglacial movements of water masses. Actually, the recorded values of periods and velocities are in good agreement with the existing concepts of short period internal gravity waves.

It was shown in works (3, 11, 14), that periods T of the progressive free internal gravity waves are within the following range in a horizontally homogeneous, rotating sea whose water is initially still

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$$T_I < T < T_m,$$

where $T_I = \frac{2\pi}{N_{max}}$ - period of free oscillations of liquid drops that belong to the level of highest stability of the water column (Vyaisyal'.); (Translator's note: "drops" is used instead of "particles".)

$N = \sqrt{gQ}$ — Vyaisyal'-Brent frequency;

g - acceleration of gravity;

$Q = -\frac{1}{\rho} \cdot \frac{d\rho}{dz}$ — Sverdrup-Hasselberg stability;

ρ - density of sea water;

z - vertical coordinate running upward;

$T_м = \frac{\pi}{\omega \sin \varphi}$ — period of inertial oscillations;

ω — angular velocity of the Earth's rotation;

φ — geographical latitude.

Period T_m is about 12 hours for the near-polar regions.

According to the calculations of P. Groyen (3), T_I can change from several seconds to tens of minutes under real marine conditions. Specifically, the Q_{max} values that lie in the range of $1 \cdot 10^{-4} \frac{1}{1} - 1 \cdot 10^{-6} \frac{1}{M}$ correspond to the periods of internal waves shown in Figure 3. Such stability values of the water masses actually characterize the central part of the Arctic Ocean (9).

The velocities of free internal gravity waves are in the range of 30 - 200 cm/sec (6). The phase velocities of waves recorded using the above-indicated apparatus mounted on a pack ice floe were in fact within this range (see Table 1). They cause oscillations of this surface with periods and phase velocities that are characteristic of internal free gravity waves. However, free surface gravity waves simultaneously exist /113 on the sea surface. With the same length as the internal waves, these have a significantly higher amplitude and much greater velocity. Therefore, wave motion that results from a combination of both classes of waves is observed on the sea surface. Vs. Berezkin noted: "It is impossible separately to observe them and to estimate the amplitudes of each" (1, p. 64). This remark pertained to the open sea. Generally speaking, it is unknown whether this will occur in the presence of an ice cover. The fact of the matter is that there is no theory of internal waves in an ice-covered sea. The authors are only familiar with a single work (2) where the effect of the ice cover on forced internal waves that arise under the effect of the periodic system of pressures in a two-layer sea was investigated. No works devoted to the theory of free internal gravity waves in an ice-covered sea could be found.

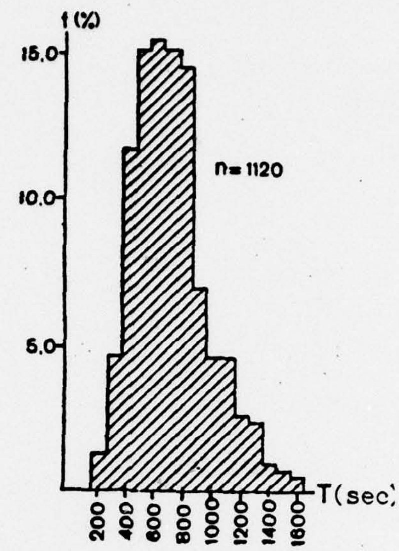

Figure 3. Histogram of internal wave periods.

In this regard it is expedient theoretically to examine free internal waves in the presence of an ice cover based on the example of the simplest model of density stratification in order to compare the theoretical results with the experimental data that were described above.

We shall introduce a cartesian system of coordinates into the examination with an axis z running upward, an axis x running to the east, and an axis y running to the north.

The equations of motion and continuity, and the condition of isopycnicity of motion shall be written in the following form:

$$\left.\begin{aligned} D\frac{d\vec{V}}{dt}+\nabla\Pi+D\nabla\Phi=0;\\ \nabla\times\vec{V}=0;\ \frac{dD}{dt}=0;\end{aligned}\right\} \tag{1}$$

where $\vec{V}=U\vec{i}+V\vec{j}+W\vec{k}\equiv(U;\ V;\ W)-$ velocity vector;

$\vec{i},\ \vec{j},\ \vec{k}-$ unit vectors that correspond to the orthogonal axes of coordinates x, y, a;

D - density;

t - time;

P - pressure;

Θ - gravity potential, $\nabla\Theta = (0, 0, g)$;

$\nabla = \left(\frac{\partial}{\partial x}; \frac{\partial}{\partial y}; \frac{\partial}{\partial z}\right)$ — gradient operator;

X - scalar multiplication sign.

We shall use the perturbation method to investigate internal waves. /114

We shall assume that a state of rest is the initial state of equilibrium of the liquid. In the state of rest the hydrodynamic parameters of the liquid depend solely on the vertical coordinate.

Let perturbation in the liquid be caused by a progressive free internal wave. Then the velocity vector, pressure, and the density of resultant motion will be

$$\vec{V} = \vec{V}'; \quad \Pi = P + p'; \quad D = \rho + \rho'. \qquad (2)$$

$\vec{V}' = (u', v', w')$ — velocity vector of the time variable perturbation;

P and ρ - pressure and density in the initial equilibrium state;

p' and ρ' - local unsteady perturbation changes of pressure and density.

We shall assume that the hydrodynamic characteristics of perturbed motion (2) differ from the corresponding characteristics of unperturbed motion by such small increments that their mutual (one word cut-off on margin and untranslatable) and squares can be ignored (translator's note: words cut-off at margin) with the value of the increments themselves. Moreover, we shall assume that not only the perturbations themselves are small, but also their derivatives along the coordinates.

We note that system (1) is reduced to a static equation in the absence of perturbations:

$$\frac{dP}{dz} = -g\rho. \qquad (3)$$

By substituting expression (2) in (1), linearizing, and using equality (3), we shall arrive at a system that describes the internal waves:

$$\left.\begin{aligned} \rho\frac{\partial\vec{V}'}{\partial t} + \nabla p' + \rho'\nabla\Phi = 0; \\ \nabla \times \vec{V}' = 0; \\ \frac{\partial\rho'}{\partial t} + \vec{V}' \times \nabla\rho = 0. \end{aligned}\right\} \qquad (4)$$

We assume that solutions to this system of equations exist which are waves of the type

$$r' = r(z)\exp[im(\nu t - x\sin\alpha - y\cos\alpha)]. \qquad (5)$$

r' - any of the values u', v', w', p', ρ';

r - amplitude of the corresponding value;

$n = \frac{2\pi}{\lambda}$ - wave number;

v - phase velocity of the internal wave;

α - angle between the positive direction of axis y and the propagation direction of the wave, read clockwise (wave azimuth); /115

$i = \sqrt{-1}$.

Using equality (5), we reduce system (4) to one ordinary linear homogeneous differential equation of the second order relative to the amplitude of the vertical velocity of internal waves:

$$\frac{d}{dz}\left(\rho\frac{dw}{dz}\right) - \rho\left(m^2 - \frac{N^2}{c^2}\right)w = 0. \qquad (6)$$

Equation (6) was first obtained by J. Fjeldstad (13).

We shall subsequently limit ourselves to the assumption that the sea consists of two homogeneous layers. The origin of the coordinates lies on the unperturbed interface of these layers. Then equation (6) is valid for each layer separately and can be written in the form

$$\frac{d^2w_n}{dz^2} - m^2w_n = 0. \qquad (7)$$

Here and below, index n = 1 corresponds to the upper layer of the sea and n = 2 - to the lower.

When integrating the system of equations (7), its solution should satisfy the boundary conditions. These conditions should be formulated on the bottom of the sea, its surface, and the internal interface where density changes abruptly.

We shall consider the sea to be horizontally infinite and therefore no conditions should be imposed on the horizontal boundaries (shores). Such formulation of the problem in the theory of internal waves is valid when shore reflection is not taken into account. The theoretical results obtained here are only applicable to the types of wave motions not directly affected by shores.

Assuming that the sea bottom is horizontal, we require satisfaction of the condition

$$\omega_2(-h_2) = 0, \tag{8}$$

where h_2 is the thickness of the lower homogeneous layer.

A kinematic condition which expresses the fact that the interface consists of identical particles and a dynamic condition which expresses the continuity of pressure during the passage through this surface should be satisfied on the internal interface.

The kinematic condition on interface is written in the following form

$$\omega_1(0) = \omega_2(0). \tag{9}$$

We shall introduce the value of displacements of liquid drops (particles) ξ_n' from the equilibrium position caused by the passage of the internal wave . These displacements are related to the vertical velocities ω_n' by the relationships

$$w_n' = \frac{\partial \xi_n'}{\partial t}. \tag{10}$$

Assuming that the unperturbed density distribution is hydrostatic and stable, i.e., $\rho_2 > \rho_1$, the dynamic condition on the interface is written in the following way:

$$p_2'(0) = p_1'(0) + g\xi_2'(0)\,\Delta\rho, \tag{11}$$

$\Delta\rho = \rho_2 - \rho_1$.

It follows from system (4) and assumption (5) that

$$p_n = -\frac{i\rho_n c}{m} \cdot \frac{dw_n}{dz}. \tag{12}$$

According to expressions (10) and (5),

$$\xi_n(z) = -\frac{i}{\sigma} w_n(z), \tag{13}$$

(translator's note: unknown mathematical symbol cut-off at margin) = cm - cyclic frequency of the internal wave.

Using relationships (12) and (13), we write equality (11) in the form

$$\gamma^2\left(\rho_2 \frac{dw_2}{dz} - \rho_1 \frac{dw_1}{dz}\right) = g\,\Delta\rho w_2 \tag{14}$$

(translator's note: one mathematical symbol cut-off at left margin) = 0.

In the absence of external forces, the equilibrium equation of the ice floe under the effect of the forces of elasticity and inertia and the bouyancy force of hydrodynamic pressure on the lower surface of the ice due to the presence of small oscillations in the liquid can be written in the following form

$$\beta \nabla_{xy}^4 \xi_1' + \varkappa \frac{\partial^2 \xi_1'}{\partial t^2} + g\rho_1 \xi_1' = p_1' \tag{15}$$

$= h_1$,

$_{xy} = \vec{i} \frac{\partial}{\partial x} + \vec{j} \frac{\partial}{\partial y}$;

h_1 - thickness of the upper homogeneous layer;

$= \frac{Eh_0^3}{12(1 - v^2)}$ - cylindrical flexural rigidity of the ice sheet;

E - modulus of normal ice elasticity;

ν - Poisson coefficient for the ice;

h_0 - thickness of ice cover;

$\chi = \rho_0 h_0$;

ρ_0 - density of the ice.

We write equation (15) in the following form, using expressions (5), (10), (12), and (13) /117

$$V^2 \frac{dw_1}{dz} - \left(g + \frac{\beta m^4}{\rho_1} - \frac{\varkappa \sigma^2}{\rho_1}\right) w_1 = 0 \tag{16}$$

when $z = h_1$.

In the absence of an ice cover ($\beta = \chi = 0$), boundary condition (16) becomes a Lamb condition - the condition of constancy of pressure on a free surface (5). Actually, in the presence of small perturbations in the liquid the condition of pressure constancy on the free surface is described in the following way:

$$\frac{\partial p_1'}{\partial t} + w_1' \frac{dP_1}{dz} = 0 \text{ when } z = h_1. \tag{17}$$

Condition (17) is transformed on the basis of equalities (3), (5), and (12):

$$V^2 \frac{dw_1}{dz} - g w_1 = 0 \text{ when } z = h_1. \tag{18}$$

Hence, boundary condition (16) is a generalization of boundary condition (18) to the case of the presence of an ice cover.

Consequently, one determines the characteristics of free oscillations in a two-layer sea in the presence of an ice layer by solving the system of equations (7) with boundary conditions (8), (9), (14), and (16).

The general solution to each equation of system (7) is written in the following way:

$$\omega_n = A_n \operatorname{ch} mz + B_n \operatorname{sh} mz, \tag{19}$$

where A_n, B_n are random constants.

By substituting equality (19) in expression (9), we obtain

$$A_1 = A_2.$$

Inasmuch as functions (19) should satisfy boundary conditions (8), (14), (16), we arrive at a homogeneous system of algebraic equations relative to coefficients A_1 and B_n:

$$\left.\begin{aligned} A_1 \operatorname{ch} mh_2 - B_2 \operatorname{sh} mh_2 = 0; \\ -A_1 \frac{g\Delta\rho}{mv^2} - \rho_1 B_1 + \rho_2 B_2 = 0; \\ A_1\left[mv^2 \operatorname{sh} mh_1 - \left(g + \frac{\beta m^4}{\rho_1} - \frac{\varkappa\sigma^2}{\rho_1}\right)\operatorname{ch} mh_1\right] + \\ + B_1\left[mv^2 \operatorname{ch} mh_1 - \left(g + \frac{\beta m^4}{\rho_1} - \frac{\varkappa\sigma^2}{\rho_1}\right)\operatorname{sh} mh_1\right] = 0. \end{aligned}\right\} \tag{20}$$

In order for a nontrivial solution to system (20) to exist, it is vital and sufficient that its determinant be zero. Thence we obtain the biquadratic equation of relative phase velocity v /118

$$\begin{gathered} m^2 v^4 \left[\rho_1 \operatorname{cth} mh_1 (\rho_2 \operatorname{cth} mh_2 + m\varkappa) + (\rho_1^2 + \rho_2 m\varkappa \operatorname{cth} mh_2)\right] - \\ - mv^2 \left[\rho_1 \operatorname{cth} mh_1 (g\rho_2 + \beta m^4) + \rho_2 \operatorname{cth} mh_2 (g\rho_1 + \beta m^4) + g\Delta\rho m\varkappa\right] + \\ + g\Delta\rho (g\rho_1 + \beta m^4) = 0. \end{gathered} \tag{21}$$

Equation (21) is transformed to the following form in the absence of an ice cover

$$n^2 v^4 (\rho_2 \operatorname{cth} mh_1 \operatorname{cth} mh_2 + \rho_1) - m\rho_2 g v^2 (\operatorname{cth} mh_1 + \operatorname{cth} mh_2) + g^2 \Delta\rho = 0. \tag{22}$$

Equation (22) is usually used to determine the velocity of internal and surface waves in a two-layer nonrotational sea in the presence of a free surface (5).

If $\rho_2 = \rho_1$ (i.e., $\Delta\rho = 0$) is placed in equation (21), then we shall arrive at the following relationship:

$$\sigma^2 = \frac{\beta m^4 + g\rho_1}{\rho_0 h_0 + \frac{\rho_1}{m \operatorname{th} mH}}, \tag{23}$$

where $H = h_1 + h_2$.

Equality (23) is the dispersion equation for elastic gravity surface waves propagating in an ice cover floating on the surface of a homogeneous sea with a depth H (10, 12, 17).

Hence, equation (21) makes it possible to determine the velocities of both surface free elastic gravity waves and internal ones. In this connection it can be viewed as a generalization of the dispersion equation for surface elastic gravity waves in the case of a two-layer sea or as a sea for a case when there is an ice cover.

We shall not fully analyze equation (21) here. We shall only analyze two boundary cases.

Let $mh_n > \frac{3\pi}{2}$. Then one can assume with high accuracy that

$$\operatorname{cth} mh_n = 1.$$

In the given case, equation (21) will look like this

$$\begin{aligned} i^2 v^4 (\rho_2+\rho_1)(\rho_1+m\varkappa) - mv^2 [\rho_2(g\rho_1+\beta m^4)+\rho_1(g\rho_2+\beta m^4)+g\,\Delta\rho\, m\varkappa] + \\ + g\,\Delta\rho\,(g\rho_1+\beta m^4)=0. \end{aligned} \tag{24}$$

The roots of equation (24) are determined by the equalities

$$v^2 = \frac{g\rho_1 + \beta m^4}{m(\rho_1 + m\varkappa)} \tag{25}$$

$$v^2 = \frac{g\,\Delta\rho}{m(\rho_2 + \rho_1)}, \tag{26}$$

the first of which makes it possible to determine the velocity of the surface waves, and the second that of the internal ones. /119

In the absence of an ice cover, we obtain

$$v^2 = \frac{g}{m}. \tag{27}$$

from relationship (25).

Equality (27) makes it possible to determine the phase velocity of free gravity waves propagating on the surface of a deep homogeneous sea.

Hence, the phase velocities of short waves in an ice-covered two-layer

sea can be found according to formulas (25) and (26). In this case the surface elastic gravity waves propagate at the same velocity as in the absence of an internal interface and the internal gravity waves propagate at the same velocity as in the absence of an ice cover.

We shall now assume that $mh_1 < 0.16\pi$, and $mh_2 > \frac{3\pi}{2}$. Then one can assume with a high degree of accuracy that

$$\mathrm{cth}\, mh_1 = \frac{1}{mh_1};\ \mathrm{cth}\, mh_2 = 1.$$

In the examined partial case equation (21) is transformed to the form as follows

$$m v^4 [\rho_1(\rho_2 + m\varkappa) + mh_1(\rho_1^2 + \rho_2 m\varkappa)] - v^2 [\rho_1(g\rho_2 + \beta m^4) + \\ + \rho_2 m h_1 (g\rho_1 + \beta m^4) + g\Delta\rho m^2 \varkappa h_1] + g\Delta\rho h_1 (g\rho_1 + \beta m^4) = 0. \tag{28}$$

If the thickness of the ice is less than the thickness of the upper homogeneous layer, then

$$\left.\begin{aligned} \rho_1^2 &\gg \Delta\rho m^2 \varkappa h_1; \\ \rho_1\rho_2(1 + mh_1) &\gg \Delta\rho m^2 \varkappa h_1 \end{aligned}\right\} \tag{29}$$

Taking into account inequalities (29), the approximated roots of equation (28) can be written in the following way:

$$v_{in}^2 = \frac{g\Delta\rho h_1 q}{\delta\gamma}; \tag{30}$$

$$v_{sur}^2 = \frac{\gamma}{m\delta} - v_{in}^2 \tag{31}$$

where v_{in} - phase velocity of internal wave in the presence of an ice cover (internal elastic gravity waves);

v_{sur} - velocity of propagation of surface elastic gravity waves in the presence of an internal interface;

$$\delta = \rho_1(\rho_2 + m\varkappa) + mh_1(\rho_1^2 + \rho_2 m\varkappa);$$
$$\gamma = g\rho_1\rho_2(1 + mh_1) + \beta m^4(\rho_1 + \rho_2 mh_1);$$
$$q = \beta m^4\delta + g\rho_1[\rho_2(\rho_1 + m\varkappa) + mh_1(\rho_1^2 + \rho_2 m\varkappa)].$$

In the absence of an ice cover, we obtain the following from expression (30) /120

$$v^2 = \frac{g\Delta\rho h_1}{\rho_2(1 + mh_1)}, \tag{32}$$

i.e., the known formula for determining the velocity of free internal gravity waves propagating along the interface of a thin surface layer and an infinitely deep lower layer.

When $\rho_1 = \rho_2$, formula (25) follows from equality (31).

The phase velocities determined according to the formulas (30) and (31), and calculated using relationships (32) and (25) are practically identical for sufficiently long (with respect to the thickness of the upper layer) waves.

For example, when $\rho_1 = 0{,}1025 \frac{\text{т} \cdot \text{v}^2}{\text{м}^4}$; $\rho_2 = 0{,}1029 \frac{\text{т} \cdot \text{v}^2}{\text{м}^4}$;

$$\rho_0 = 0{,}090 \frac{\text{т} \cdot \text{v}^2}{\text{м}^4}; \quad g = 10 \text{ м/v}^2; \quad E = 7{,}9 \cdot 10^5 \text{ т/м}^2;$$

$$\nu^1 = 0{,}28; \quad h_0 = 4{,}5 \text{ м}; \quad h_1 = 50 \text{ м};$$

$$h_2 = 3000 \text{ м}; \quad m = 8{,}61 \cdot 10^{-3} \frac{1}{\text{м}};$$

the velocities of the internal waves are

$$v_{in} = \pm 1.17 \text{ m/sec}, \tag{33}$$

according to formula (32), and the velocities of surface waves are

$$v_{sur} = \pm 33.76 \text{ m/sec}, \tag{34}$$

according to formula (25).

Phase velocities v_{in} and v_{sur} calculated on the basis of relationships (30) and (31) with the same numerical values of the parameters slightly differ from the found values (33) and (34).

Hence, long surface waves in a two-layer sea covered with ice propagate at nearly the same velocity as in the absence of an ice cover and an internal interface. For example, with the above-indicated numerical values of free parameters, the velocities of surface waves calculated according to formula (27) are

$$v = \pm 34.08 \text{ m/sec}.$$

These values of v differ little from the calculated ones (34).

The phase velocities of long internal waves in an ice-covered sea are also independent of the characteristics of the ice cover.

We shall determine the value of the ratio of the amplitude of an elastic gravity wave on the surface of the sea $\xi_1(h_1)$ to the amplitude of an internal wave $\xi_2(0)$. According to (13) and (19), we have

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[1]The numerical values of E and ν were directly determined while making seismic observations of oscillations of the ice cover.

$$\frac{\xi_1(h_1)}{\xi_2(0)} = \operatorname{ch} mh_1 + \frac{B_1}{A_1} \operatorname{sh} mh_1. \tag{35}$$

By substituting the value of the relationship $\frac{B_1}{A_1}$ in equality (35), determined by the third equation of system (20), we find /121

$$\frac{\xi_1(h_1)}{\xi_2(0)} = \frac{mc^2}{mc^2\left(\operatorname{ch} mh_1 + \frac{\varkappa m}{\rho_1} \operatorname{sh} mh_1\right) - \left(g + \frac{\beta m^4}{\rho_1}\right) \operatorname{sh} mh_1}. \tag{36}$$

In the absence of an ice cover, the classical formula for the ratio of the amplitude of the surface gravity wave to the amplitude of the internal wave follows from the relationship (36) (5):

$$\frac{\xi_1(h_1)}{\xi_2(0)} = \frac{mc^2}{mc^2 \operatorname{ch} mh_1 - g \operatorname{sh} mh_1}. \tag{37}$$

We note that $\xi_1(h_1)$ in expression (36) should be defined as the deflection amplitude of the ice sheet and in (37) - the amplitude of surface free gravity waves.

Wavelength was assumed to be 730 m for the numerical calculations above. The velocity of the internal wave (33) calculated with this wavelength value agrees entirely satisfactorily with the experimentally obtained velocity (see Table 1, example 2). This wave can be viewed as quite long with respect to the depth of the upper homogeneous layer of the sea. The amplitude of the examined wave on the sea surface, according to observations made using tilt meters, is $\xi_1(h_1) = 218 \cdot 10^{-5}$m. According to formula (36), on the interface the amplitude of this wave reaches a value of $\xi_2(0) = -0.83$ m. The minus sign indicates that oscillations of the sea surface and the internal interface are in opposite phase.

Hence, the amplitude of oscillations of the interface was slight for the displacements of the ice cover obtained in the process of observations. This very circumstance is possibly why hydrological observations conducted in parallel with recording ice oscillations have not made it possible to establish the characteristics of internal waves.

The results of ~~the~~ full-scale observations ~~presented above~~ and ~~the~~ theoretical calculations ~~that were made~~ provide a basis to assume that two classes of free oscillations can simultaneously appear in a heterogeneous sea in the presence of an ice cover: internal ones and surface elastic gravity waves. In this case the internal waves cause oscillations of the ice cover which can be recorded ~~in principle~~ using the appropriate apparatus.

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